Persistent Blessings of Luck:
Capital and Deal Flows in Venture Investment[*]

Lin William Cong[†]   Yizhou Xiao[§]

December 2016

[Click here for most updated version]

Abstract

Persistence in fund performance in the private equity industry, especially venture funds, is often interpreted as evidence of differential abilities among the managers. We present a dynamic model of delegated investments that produces performance persistence and predictability without investment skill heterogeneity. Capital and quality projects exhibit strong complementarity, and endogenously flow to recently successful funds due to incentivization of managerial effort with continuation value and assortative matching. Initial luck therefore has an enduring impact on fund performance and managerial compensation, and investors benefit from working with multiple funds with tiered contracts. Consistent with empirical findings, our model also predicts that venture funds that persistently outperform encourage greater innovation and attract better entrepreneurial projects using seemingly less favorable contract terms.

JEL Classification: G10; L26; O31; G24
Keywords: Private Equity, Venture Capital, Innovation, Delegated Investment, Performance Persistence, Fund Manager Skills

1 Introduction

Copious literature asks whether investment managers differ in skills. Studies of individual stocks, mutual funds, and other class of funds generally find that investors do not consistently outperform the market and fund performances are not persistent. An important exception is the private equity (PE) industry, most notably venture capital (VC) and buyout (LBO) funds. Kaplan and Schoar (2005) show in their seminal study that PE firms typically manage sequences of funds, and the performance of one fund predicts the fund flow and performance of the subsequent fund. Besides the fund performance persistence, Hsu (2004) finds that firms are more likely to accept an offer, even if the terms are less attractive, from a VC with good past performances. A widely-adopted interpretation of such performance persistence and entrepreneur funding choice is that PE managers differ in their abilities, and the more skilled ones consistently outperform the others.

We present a new theory to challenge and complement the conventional wisdom. The key insight is that in the PE industry, deal sourcing is important and different funds endogenously experience differential deal flows. If a manager is lucky in the current fund, he may find it easier to raise the next fund with more favorable terms, which in turn allows them to be more tolerant towards failures and experimentation, attracting better deals that perpetuate his good performance. This positive reinforcement can lead to persistence in differential performance across managers even when they do not differ in skills. In spirit, this paper is akin to Berk and Green (2004): they argue that the lack of persistence in returns does not necessarily mean differential ability across managers is non-existent or unrewarded; we argue that performance persistence and entrepreneur funding choice do not necessarily imply heterogeneity in managerial skills. In fact, luck could have an enduring impact on fund performance and managerial compensation due to the strong complementarity between endogenous capital and deal flows.

Given both persistent fund performance and entrepreneur's preference on top VC fund offers are widely observed and interpreted as evidence of differential manager skills, we add to the literature by underlying that this connection is more subtle and providing an alternative framework for thinking about delegated investment, emphasizing the crucial roles of deal

1

flows and their interaction with capital flows. To be clear, our model does not contradict that in reality, managers at VC and PE funds probably do have differential skills. On the contrary, we illustrate that the complementarity between capital flow and deal flow can drastically augment the effect of small skill differences, therefore contributing to the existing literature linking persistence in fund performance to differential manager skills.

While we believe our intuition and economic mechanism apply more broadly, we cast the theory in the context of VC industry for several reasons. First, entrepreneurship is an important engine for innovation and VC plays an important role in supporting the formation of innovative enterprises. The companies funded by venture capital are a major part of the U.S. economy and the US VC ecosystem deployed US\$58.8 billion in 2015 alone.[1] In developing economies with inefficient banking system, VC also proves crucial in financing productive young firms.[2] The sheer size of the industry and its significance in motivating innovation warrant careful analysis. Yet despite the emerging empirical research, the theoretical underpinnings are understudied. Second, endogenous deal flows and persistence in fund performance are salient features in the VC industry. Funds do not get to observe all potential deals due to limited attention and high search cost, and rely on the funds' visibility, PR, network, reputation, track record, and affiliations with incubators, proximity to geographical focal point (Silicon Valley), and presence in online platforms. Third, due to the early-stage nature of projects, VC industry is plagued by moral hazard and adverse selection, and studying the contracts in the VC industry, implicit or explicit, adds to our understanding of contract theory in general.

Specifically, we introduce a new model of delegated investment by homogeneous managers that produces a stationary equilibrium with endogenous capital and deal flows, persistent fund performance, entrepreneur's funding-offer preference, and a number of other implications consistent with real life observations. Though we specialize to the agency friction of moral hazard, we illustrate that the same mechanism of capital and deal flows would apply

---

[1] According to the MoneyTree Report from PricewaterhouseCoopers LLP (PwC) and the National Venture Capital Association (NVCA), based on data provided by Thomson Reuters.

[2] For example, China raised about a whooping US\$338 billion in venture capital in 2015. The fundraising involves both government-backed and private ventures, according to data compiled by the consultancy Zero2IPO Group.

to the case of asymmetric information or symmetric learning. In our main setup, a group of entrepreneurs are born with projects in each period and seek financing and value-added service from VCs. A mediocre project succeeds with some probability and pays off a mediocre amount, but a good project benefits from experimentation and yields (with a potentially smaller probability) a payoff so large that it should be nurtured with innovative technology. VCs select projects and decide on (1) whether to use an innovative or conventional nurturing strategy and (2) whether to exert effort to improve the chance of project success. Investors invest in VCs, using both a share of the fund profit and implicit capital commitment for the next fund operated by the same VC to motivate effort by VCs. In equilibrium, investors offers two types of contracts, one encouraging VCs to use innovative nurturing and the other, conventional nurturing. By using promotion from conventional contract to innovative contract, demotions from innovative contract to conventional contract, or terminations of contracts, investors can incentivize efforts by managers to improve project quality. The set of incentive contracts used by investors exhibit hierarchical structure when good project's success probability is low relative to agents' discount rate, which is often the case in the PE industry.

We first establish that capital and deal endogenously flow to fund managers that are successful due to luck. Investors implicitly commit future capitals as a way to motivate managers' effort, and improve contract terms to be more tolerant towards experimentation and innovation. This arrangement attracts entrepreneurs with innovative projects. In the equilibrium, the complementarity between capital and deal flows implies that under investor's optimal contract, fund performance and investor returns are persistent and predictable, and entrepreneurs are willing to accept offers from VC funds with more tolerant contracts. Unlike performance persistence caused by skill heterogeneity, better-performing managers do not increase fee to make after-fee returns unpredictable due to competition from other managers (none of them has superior skills than others). In addition, managers at lucky funds persistently get higher compensations.

We also find that top-performing funds are more tolerant towards experimentation and encourage greater innovation. The intuition is that investors cannot disentangle luck and

managers' effort, and use continued funding to motivate managers to put in effort. Due to their luck in the past, some managers are rewarded better contract terms that reduce the pressure to produce quick but mediocre results, and allow them to be more innovative. Therefore they take high risk and fully exploit the benefit of the positive skewness in startup payoff distribution. Moreover, to the extent that project quality is private information to the fund managers or the entrepreneurs, matching up with a successful fund signals project quality through selection, and top-performing funds have an endogenous certification effect even though managers do not have differential skills.

This paper foremost contributes to our understanding of managerial skills and fund performance. Observing the lack of return persistence in mutual fund, Berk and Green (2004) illustrate that no performance persistence does not necessarily mean no skill difference. Motivated by the empirical evidence of persistence in VC fund performance (Kaplan and Schoar, 2005; Phalippou and Gottschalg, 2009; Harris, Jenkinson, Kaplan, and Stucke, 2014; Ewens and Rhodes-Kropf, 2015), Glode and Green (2011) and Hochberg, Ljungqvist, and Vissing-Jorgensen (2014) argue incumbent investors with soft information concerning general partners' skill can hold up the manager and extract information rents. Marquez, Nanda, and Yavuz (2015) suggest that VC's excess effort to manipulate entrepreneur's beliefs about his ability also leads to persistence. Acharya, Gottschalg, Hahn, and Kehoe (2013) analyze deal-level data and show that managers with disparate backgrounds add value in different deals. Gompers, Kovner, Lerner, and Scharfstein (2006) argue that the fact some VCs can select successful serial entrepreneurs is evidence for VC skill. Sorensen (2007) also explain VC performance persistence using heterogeneity in deal flows caused by heterogeneity in skills. Unlike these papers with results and intuition predicated on managers' having sufficiently differential skills, our paper shows luck alone can give rise to performance persistence. We are not contradicting the existing literature by claiming managers do not have differential skills, but aim to provide a cautionary tale that existing empirical evidence is insufficient proof of significant differences in skill.

This paper further adds to the growing literature on the VC industry. Starting from Gompers (1996), a number of studies examine the grandstanding by VC funds. Megginson

and Weiss (1991) document the certification role of VC in IPOs to mitigate information asymmetry. Tian and Wang (2014) find that firms backed by more failure-tolerant VC investors are significantly more innovative. Hsu (2004) finds that firms are more likely to accept an offereven if the terms are less attractivefrom a VC with good past performances. Our model adds theoretical foundations for these phenomena without resorting to VC fund GP heterogeneity.

This paper is also related to delegated investment and contracts that motivate innovation. The investors and fund managers' implicit contract can affect innovation. More importantly, we highlight the complementarity of capital flows and deal flows, which to our knowledge is realistic yet largely unexplored in the literature. Manso (2011) argues that the optimal way to motivate innovation is to show tolerance for failure. Our paper shows that this tension links the investors and portfolio companies. Bolton and Scharfstein (1990) use funding termination to mitigate managerial incentive problems. They also show that withholding future funding can play a role similar to demanding repayment in forcing liquidation. While this is often applied to interpret the implicit ex post staging of finance for entrepreneurs, it can be used to analyze the long-term financing of VCs. Stiglitz and Weiss (1983) provide conditions for and characterize equilibrium contingency contracts with potential terminations of relationship. Their argument about the credible threat to abandon a venture, even when the firm might be economically viable, is the key to the relationship both between the entrepreneur and the venture capitalist (staged financing), and between general and limited partners of a fund (long-term financier for sequential funds). Finally, our model exhibits features of "limits of arbitrage", and intermediaries may take inefficient actions in equilibrium.[3] However, the presence of deal flows mitigates such inefficiencies.

The rest of the paper is organized as follows: Section 2 lays out the basic framework and defines the equilibrium. Section 3 solves the model and characterizes the equilibrium; Section 4 talks about model implications. Finally, Section 5 concludes.

---

[3]See, for example, Shleifer and Vishny (1997).

2 Model Setup

This section sets up the model and defines the equilibrium. There are three groups of players in the economy. Time is discrete and infinite, and is labeled by $t = 1, 2, 3, \cdots, \infty$. For simplicity, all players are risk-neutral and share the same time discount rate $\beta \in (0, 1)$.

A unit measure of entrepreneurs (EN) are born in each period. A fraction ϕ of them are endowed with innovative project ideas (I-projects), the rest with conventional ideas (C-projects). Let the set of all time-t entrepreneurs be $\mathcal{E}_t$, the ones with innovative ideas $\mathcal{E}_t^I$, and the ones with conventional ideas $\mathcal{E}_t^C$. All projects require nurturing by venture capitalists and each pays X_C with probability $p_C \in (0, 1)$ under conventional technology (C-Technology). I-projects also benefit from aggressive and innovative nurturing technology (I-Technology) under which each succeeds with probability p_I and payoff X_I. The idea yields 0 if it fails. Each entrepreneur lives for one period and then permanently exits the market.[4] To capture payoff skewness in entrepreneurial endeavors, we assume project payoffs are skewed and I-projects are scarce

Assumption 1

$$p_C \geq p_I \quad and \quad p_C X_C < p_I X_I, \tag{1}$$

and

Assumption 2

$$\phi \leq \frac{p_C}{1 + p_C - p_I}, \tag{2}$$

which are non-restrictive. For example, these assumptions hold when innovative projects pay off more upon success and are rare enough that the measure of failed I-projects is still less than that of successful C-projects.

There is an infinitely supply of venture capitalists aspirant of becoming general partners of VC funds (GPs). In each period, after successful fundraising, a GP $g \in G_t$ screens the

---

[4]In other words, each entrepreneur only interacts with venture capitalists once, which captures reality in a reduced-form: while serial entrepreneurs exist, they are rare and their contracts with financiers are based on individual projects; startups not immediately funded are often out-competed by rivals.

type of projects, ascertains project types, and for simplicity is assumed to contract with one entrepreneur (each fund only operates one project). GPs choose either I- or C-technology based on their contracts with the investors.[5] They also decide whether to incur effort cost $e > 0$ to augment the project's success probability by $0 < \Delta < \frac{1-p_C}{p_C}$.

In the next period, the fund's investment outcome is realized. The GP pays the entrepreneur and investors, raises capital for the next fund and closes the initial fund. If a GP fails to raise capital for a fund, he permanently exits the market of new funds.[6] For simplicity we assume the GPs contract with entrepreneurs to share the project payoff in fixed proportions $\rho : 1 - \rho$, $\rho \in (0, 1)$.[7] Denote g's decision on nurturing technology by $n \in \{C, I\}$. Then a GP's strategy has $\Lambda_g = (\mathbb{I}_e, n, \rho)$ where $\mathbb{I}_e$ is the effort indicator, and $1 - \rho$ is the share of project payoff to the entrepreneurs. It is worth emphasizing that in our model GPs provide value-added services because they screen, improve, and nurture projects, but they do not have differential skills.

There is one representative investor who can invest in multiple funds each period as a limited partner (LP). She represents university endowments, pension funds, family offices, etc, and has deep pockets to finance all potential projects. In each period t, LP chooses to invest a set of $F_t = [0, \zeta]$, $\zeta \in \mathcal{R}_+$ funds and decides her investment plan $A_t(f)$, $f \in F_t$. Then the time t set of GPs successful at fundraising is $G_t = \{g | \exists f \in F_t, \ s.t. \ g = A_t(f)\}$. For each fund she works with, she offers a contract explicit on the ownership shares of the fund and either explicit or implicit on whether and how to continue working with the GP contingent on the current fund performance.

We are essentially assuming that the LP has limited commitment power in that she can only commit to short-term contracts specifying the profit split for the current fund and the contract extension policy contingent on the fund's performance. This assumption captures reality reasonably well for two reasons: First, our setup can be micro-founded by overlapping generations of finitely-lived venture capitalists, which we illustrate in the

---

[5]Nurturing can be interpreted as experimentation jointly with the entrepreneurs. GPs in general can influence firm operations, team building, and experimentation styles by setting different contract terms, taking hidden actions, showing different attitudes towards failures etc.

[6]Failed GPs seldom manage to raise another fund, at least for a long period of time if at all.

[7]We later endogenize the contract terms between GP and EN in a Nash bargaining game.

appendix with GPs that live for three periods. Compared to the long-lived LP such as university endowments, pension funds, family offices, etc., GPs have relatively short career horizon with finite life-cycle of VCs and high career turnover. This reality renders long-term contracts or contingencies on fund performance over multiple periods unnecessary.

Second, if the LP has full commitment power, then similar to the large literature on dynamic contracting, a long-term contract with a GP often entails a back-loaded compensation plan (using only continuation value without cash payments) at the early stage of the contract, which is inconsistent with real-life observations that GPs receive cash payments for each fund they operate. This contradiction corroborates our assumption of the LP's limited commitment power on long-term contracts.[8]

The LP thus offers contracts in equilibrium of the form $\Phi_g = \{\alpha, n, V_f, V_s\}$, where α is GP g's total share of the fund's return, and V_S, $S = \{s, f\}$ is g's promised continuation value given the fund's project outcome and all agents' equilibrium strategies. By contracting on the project payoffs, the LP can dictate whether a GP uses I-technology (I-contract) or C-technology (C-contract).[9] The LP's strategy is consistent because for each $g \in G_{t-1}$, his time t expected utility equals the implicit promised value in his time $t-1$ contract given his time $t-1$ fund performance. Without loss of generality, at any time t if g has not started a fund yet or he fails to raise a consecutive round of fund, the offer is denoted by $\Phi_g = 0$. We further assume that

Assumption 3

$$\beta \Delta p_C \rho X_C \geq \frac{1}{\beta} - p_C(1 + \Delta))e \tag{3}$$

as shown latter, this assumption implies that GP's effort improves probability of success sufficiently much that the LP wants to incentivize effort regardless of the equilibrium innovation

---

[8] See Biais, Mariotti, Plantin, and Rochet (2007), DeMarzo and Fishman (2007), Sannikov (2008) and He (2009).

[9] The I-contract penalizes outcomes X_C to induce the GPs to use innovative technology; similarly, C-contract penalizes outcomes X_I to induce conventional technology. When project payoffs are not contractible but project successes or failures are, the LP needs to incentivize the technology choice, but our key intuition goes through and we can show under mild parameter restrictions, the contracts are the same. Another way to interpret our assumption is that I-technology and C-technology require different cash flows for the LP, say through capital call provisions, which are contractible.

technology.

While in our baseline model funds offer the same ρ to entrepreneurs, entrepreneur's endogenous fund choice is crucial because GPs use different technologies under $\{\Lambda_g\}$ given contracts $\{\Phi_g\}$. For each entrepreneur $i \in I_t$, her funding offer choice is denoted by $\Psi_i = g$, $g \in G_t$. We model project deal flows by assortative matching in each period between projects and the GPs with successful fundraising. As in the Deferred-Acceptance Algorithm for stable matching (Gale and Shapley (1962)), GPs observes entrepreneurs' types and simultaneously making offers to their top choices. In the offers, funds can either credibly post the nurturing strategy, or the valuation of the project. For example, funds can guarantee to pay the expected value of entrepreneur's payoff in the next period. Entrepreneurs reject all but their top choices of funds, and break indifference by randomizing among funds they equally prefer. Rejected funds then make the next round of offers, and the remaining entrepreneurs again reject all but their top choices. The process goes on until all the funds have projects or all the projects have VC backing.

Now we are ready to define the equilibrium.

Definition 1

A equilibrium is LP's strategy $\Xi^* \equiv \{\{\Phi_f^*\}_{f \in F_t}, A_t\}_{t=1,2...}$, GPs' strategies $\{\{\Lambda_g^*\}_{g \in G_t}\}_{t=1,2...}$ and entrepreneurs' strategies $\{\{\Psi_i^*\}_{i \in I_t}\}_{t=1,2...}$ such that:

1. *For each GP $g \in G_t$, conditional on entrepreneurs' funding offer choices $\{\Psi_i^*\}_{i \in i_t}$, LP's contract Φ_g^* and other GPs' strategies $\{\Lambda_{g'}^*\}_{g' \in G_t \backslash g}$, Λ_g^* satisfies*

$$\Lambda_g^* \in argmax_{\Lambda_g} \; E^{\Lambda_g}\{\alpha\rho R_g + \beta V_S\}; \tag{4}$$

   where R_g is the GP g's fund return, and V_S is the promised value in state $S \in s, f$.

2. *For each entrepreneur $i \in I_t$, conditional on GPs' strategies $\{\Lambda_g^*\}_{g \in G_t}$, her funding offer choice Ψ_i^* satisfies*

$$\Psi_i^* \in argmax_{\Psi_i} \; E^{\Psi_i}\{(1-\rho)R_g\}; \tag{5}$$

3. *Conditional on GPs' strategies $\{\{\Lambda_g^*\}_{g \in G_t}\}_{t=1,2...}$ and entrepreneurs' strategies $\{\{\Psi_i^*\}_{i \in I_t}\}_{t=1,2...}$,*
   Ξ^* *maximizes L's discounted expected investment profit:*

$$\Xi^* \in argmax_\Xi \ E^\Xi \{\sum_{t=1}^{\infty} \int_{F_t} \beta^t (1-\alpha) \rho R_f df\}; \tag{6}$$

In this paper we focus on dynamic equilibria that are stationary, as defined below.

Definition 2

Stationary Equilibrium of Delegated Investment:

1. *The set of funds F_t and fund contracts $\{\Phi_f^*\}_{f \in F_t}$ are time-invariant and nonrandom;*

2. *Let $M_t(\Phi, \Phi')$ be the time t measure of funds that the GP was offered a contract Φ in last period and receives a contract Φ' at current period, then $M_t(\Phi, \Phi')$ is time-invariant for all Φ and Φ'.*

In the stationary equilibrium, the aggregate distribution of funds is time-invariant and deterministic. For the LP, the stationary equilibrium is essentially static. She will finance constant measures of contracts, finances the same measure of ideas with funds that apply innovative nurturing technology over time and receives time-invariant investment return. From GPs' perspectives, the equilibrium is stationary in the sense that the aggregate measures of GPs accepting different contracts are time-invariant and deterministic.

3   Dynamic Equilibrium

Assortative Matching of Funds and Projects

Under I-technology, an entrepreneur with I-project gets $(1-\rho)(1+\Delta)p_I X_I$, which is greater than $(1-\rho)(1+\Delta)p_C X_C$ under C-technology. Hence she strictly prefers funds under I-Contracts. Moreover, accepting offers from I-contracts funds is a weakly dominating strategy even when entrepreneurs do not know the type of their ideas. Similarly, funds with I-contracts prefer I-projects. Therefore, we have a unique matching equilibrium that

is positive assortative in the sense that more innovative projects are matched with more innovative funds.

Moral Hazard and Incentive Contracts

We are interested in the case in which the LP wants to motivate effort from the GPs, and decides on the technology choice of GPs in equilibrium by offering either an I-contract or a C-contract.

To induce effort, a C-contract must satisfy:

$$\beta[(1+\Delta)p_C(V_s^C+\rho X_C\alpha^C)+(1-p_C(1+\Delta))V_f^C]-e \geq \beta[p_C(V_s^C+\rho X_C\alpha^C)+(1-p_C)V_f^C], \quad (7)$$

where α^C is the share given to the GP. Therefore, fixing V_s^C and V_f^C, the cheapest contract from the LP's perspective satisfies

$$\alpha^C = \frac{e - \beta\Delta p_C(V_s^C - V_f^C)}{\beta\Delta p_C\rho X_C} \quad (8)$$

The payoff (agency rent) for a GP under C-contract is then

$$V_{GP}^C = \beta[(1 + \Delta)p_C(V_s^C + \alpha^C\rho X_C) + (1 - (1 + \Delta)p_C)V_f^C] - e = \frac{e}{\Delta} + \beta V_f^C. \quad (9)$$

Intuitively, for the GP the value of operating a C-contract fund consists of two components. V_f^C represents the future payoff in the worst scenario, while $\frac{e}{\Delta}$ describes the additional economic rent. Since $V_f^C \geq 0$, $\frac{e}{\Delta}$ is the GP's minimal agency rent. Similarly, the incentive-compatibility for exerting effort in an I-contract requires:

$$\alpha^I = \frac{e - \beta\Delta p_I(V_s^I - V_f^I)}{\beta\Delta p_I\rho X_I}, \quad (10)$$

and the GP's rent under I-contract is then

$$V_{GP}^I = \beta[(1 + \Delta)p_I(V_s^I + \alpha^I\rho X_I) + (1 - (1 + \Delta)p_I)V_f^I] - e = \frac{e}{\Delta} + \beta V_f^I. \quad (11)$$

11

Equilibrium with Fixed Technology

To illustrate the idea, let us first examine the benchmark equilibrium where there is only one technology available. Without loss of generality, we focus on the C-contracts case.

Notice that in an equilibrium with only C-contracts, the present value to the LP is:

$$V_{LP}^C = \frac{\beta}{1-\beta}(1+\Delta)p_C(1-\alpha)\rho X_C \tag{12}$$

To maximize the LP's profit, the optimal contract minimizes α, and α can be rewritten as:

$$\begin{aligned}
\frac{\beta}{1-\beta}(1+\Delta)p_C\alpha\rho X_C =& [\frac{(1+\Delta)e}{\Delta} + \beta V_f^C] \\
& - \frac{\beta}{1-\beta}P(V_s^C,V_f^C)[\frac{(1+\Delta)e}{\Delta} + \beta V_f^C]
\end{aligned} \tag{13}$$

The LHS is NPV of future cash payments to GPs operating the fund. The first term on the RHS is the expected total value paid to the current GP to motivate effort, and the second term is the discounted future "replacement" costs of terminating the incumbent GP and paying a new GP in each period, where $P(V_s^I, V_f^C)$ is the replacement probability given the contract. Now the LP's profit is:

$$\begin{aligned}
V_{LP}^C =& \frac{\beta}{1-\beta}(1+\Delta)p_C\rho X_C - [\frac{(1+\Delta)e}{\Delta} + \beta V_f^C] \\
& - \frac{\beta}{1-\beta}P(V_s^C,V_f^C)[\frac{(1+\Delta)e}{\Delta} + \beta V_f^C]
\end{aligned} \tag{14}$$

The first term is NPV of future cash flows from the projects, the second term is the expected total value paid to the current GP to motivate effort, and the third term is the discounted future "replacement" costs. Replacements are costly because we are giving agency rent to a new GP without using it to motivate his effort in the previous period. Let's first consider two extreme cases.

Example 1 (Single Period Contract)
The LP commits to terminate the incumbent GP and paying a new GP in each period. Then
$V_s^C = V_f^C = 0.$

Given the contract, $V_{GP}^C = \frac{e}{\Delta} + \beta \times 0 = \frac{e}{\Delta}$. The LP's expected profit is:

$$
\begin{aligned}
V_{LP}^C =& \frac{\beta}{1-\beta}(1+\Delta)p_C\rho X_C - \frac{(1+\Delta)e}{\Delta} \\
& - \frac{\beta}{1-\beta}\frac{(1+\Delta)e}{\Delta} \\
=& \frac{\beta}{1-\beta}(1+\Delta)p_C\rho X_C - \frac{1}{1-\beta}\frac{(1+\Delta)e}{\Delta}
\end{aligned}
\tag{15}
$$

Under the single period contract, the low continuation value $V_s^C = V_f^C = 0$ suggests a low agency cost $\frac{(1+\Delta)e}{\Delta}$ for the incumbent GP but a high replacement rate and associated high replacement agency costs.

Example 2 (Perpetual Contract)

The LP commits to renew the contract with the incumbent GP in each period. Then $V_s^C = V_f^C = V_{GP}^C$.

Given the contract, $V_{GP}^C = \frac{e}{\Delta} + \beta V_{GP}^C = \frac{1}{1-\beta}\frac{e}{\Delta}$. The LP's expected profit is:

$$
\begin{aligned}
V_{LP}^C =& \frac{\beta}{1-\beta}(1+\Delta)p_C\rho X_C - \frac{1}{1-\beta}\frac{(1+\Delta)e}{\Delta} - 0 \\
=& \frac{\beta}{1-\beta}(1+\Delta)p_C\rho X_C - \frac{1}{1-\beta}\frac{(1+\Delta)e}{\Delta}
\end{aligned}
\tag{16}
$$

Under the perpetual contract, the high continuation value $V_s^C = V_f^C = V_{GP}^C$ suggests a high agency cost $\frac{1}{1-\beta}\frac{(1+\Delta)e}{\Delta}$ for the incumbent GP but a low replacement rate and associated low replacement agency costs 0. The following proposition states the optimal contract that balances the trade off between incumbent GP and replacement agency costs.

Proposition 1

With a single technology n available, $n \in \{I, C\}$, there is an essentially unique equilibrium. The LP offers a measure p_n of n-contracts to GPs that are recently successful, and a measure $1 - p_n$ to new GPs, all with terms:

1. *$\alpha^n = \frac{(1-\beta p_n)e}{\beta \Delta p_n \rho X_n}$.*

2. *Renewal of the same contract upon project success.*

3. Permanent termination of the current GP.

The equilibrium involves termination upon failure and contract renewal upon success. Mathematically, $V_f^C = 0$, and $V_s^C = V_{GP}^C$. Under the optimal contract, the present value to the LP is:

$$V_{LP}^C = \frac{\beta}{1-\beta}(1+\Delta)p_C\rho X_C - \frac{(1+\Delta)e}{\Delta} - \frac{\beta}{1-\beta}(1 - p_C(1+\Delta))\frac{(1+\Delta)e}{\Delta} \qquad (17)$$

Given assumption 3, $V_{LP}^C \geq 0$, and therefore all projects are funded.

Equilibrium with Two Technologies

Now we move to the case when both technologies are available, the LP can use both I- and C-contracts. The optimal contract problem is complicated in the sense that the LP may promise a mixture of different types of contracts as the continuation value and the associated contract type transition rates need to guarantee the existence of the steady state. Instead of solving the optimal contract directly, we begin our analysis with solving the optimal contract in a special scenario, and then show that this solution is also the optimal contract in our model.

Unlimited Supply of Zero Profit C-Projects

For this moment, we first consider the optimal contract problem in the following special scenario: A measure M_I of entrepreneurs (EN) are born with innovative project ideas (I-projects) in each period, there are unlimited supply of entrepreneurs with conventional ideas (C-projects). The conventional ideas payoff satisfies

$$\beta\Delta p_C\rho X_C = (\frac{1}{\beta} - p_C(1+\Delta))e. \qquad (18)$$

So the LP will be indifferent to finance or abandon C-projects under the contract characterized in proposition 1.

We start our analysis by proposing characteristics of equilibria (if one exists). Those characteristics are verified by arguing that for any steady state that does not satisfy one

of the characteristics, we can propose different contracts or a different contract allocation strategy A_t such that those changes introduce another steady state and the LP is strictly better off.

The first result simplifies our analysis in the sense that we only need to focus on a single contract for each type of projects.

Lemma 1

In equilibrium, the LP offers only one type of C-contact and one type of I-contract. Let the measure of I-contacts be m_I. In equilibrium, $m_I = M_I$.

Since the agency rent V_{GP}^C and V_{GP}^I depend on V_f^C and V_f^I, our analysis focus on GPs' continuation values upon project failures.

Lemma 2

In equilibrium, $V_f^C = 0$.

From this we know in equilibrium $V_{GP}^C = \frac{e}{\Delta} \leq V_{GP}^I$.

Lemma 3

In equilibrium, $V_f^I = 0$ if $\beta < p_I(1+\Delta)$, and $V_f^I = V_{GP}^C$ otherwise. In either case $V_s^I = V_{GP}^I$.

We now derive the stationary equilibrium. Funds with C-contracts would make offers to all ENs, but ENs with I-projects always prefer funds with I-contracts. Therefore, given a stationary deal sources distribution in each period, the law of large numbers and LP rationality imply that a measure ϕ of I-projects are financed by funds with I-contracts, and the remaining are financed by funds with C-contracts.

Because the promised continuation value comes from future cash flows, which are determined by the steady state α_{ss}^I, α_{ss}^C, and the amount of I-contracts and C-contracts. Therefore, the LP solves

$$\max_{\{\alpha_{ss}^I, \alpha_{ss}^C\}} \frac{\beta}{1-\beta} \left\{ m_C p_C (1-\alpha_{ss}^C)\rho X_C + m_I p_I (1-\alpha_{ss}^I)\rho X_I \right\} \tag{19}$$

To solve this, we first observe that in any period in the steady state, the total future cash payments to all GPs that manageing a fund now or latter can be written as

$$\frac{\beta}{1-\beta}\{m_C p_C \alpha_{ss}^C \rho X_C + m_I p_I \alpha_{ss}^I \rho X_I\}. \tag{20}$$

Hence to maximize the LP's payoff is equivalent to minimizing this total payment to GPs. On the other hand, since all promised value must be paid in future, this total payment to GPs can be written alternatively as

$$\mathcal{C} \equiv \frac{e}{1-\beta} + \frac{\beta}{1-\beta} K V_{GP}^{new} + V_{GP}^{total} \tag{21}$$

where K is the steady state replacement rates for GPs. The first term is the total effort expense incurred from now on; the second term is the present value of future payoffs given to the future new entrant GPs; the third term represents the present value to the operating funds in this period,

$$V_{GP}^{total} = m_I V_{GP}^I + m_C V_{GP}^C = m_I \left(\frac{e}{\Delta} + \beta V_f^I\right) + m_C \left(\frac{e}{\Delta} + \beta V_f^C\right) \tag{22}$$

The equilibria contracts have the following forms:

Proposition 2 (Equilibria with Hierarchical Contracts)

When the probability for the innovative project to succeed under managerial effort is small, i.e., $\beta \geq p_I(1 + \Delta)$, the LP offers a measure M_I of I-contracts to GPs who are recently successful with terms:

1. $\alpha^I = \frac{(1-\beta^2 p_I)e}{\beta \Delta p_I \rho X_I}$.

2. Renewal of the same contract upon project success and payoff X_I.

3. Continued funding under a C-contract upon project failure.

The LP offers an arbitrary measure $m_C \geq m_C^* \equiv \frac{[1-(1+\Delta)p_I]M_I}{(1+\Delta)p_C}$ of C-contracts. To be more specific, she offers a measure $m_C(1-(1+\Delta)p_C)$ of C-contracts to new GPs, $m_C(1-\lambda)(1+$

$\Delta)p_C$ to GPs who are recently successful under C-contracts, and $m_I(1 - (1 + \Delta)p_I)$ to GPs who recently failed under I-contracts, all with terms:

1. $\alpha^C = \frac{(1-\beta(1+\lambda\beta)p_C)e}{\beta\Delta p_C \rho X_C}$.

2. Upon project success and payoff X_C, continued funding with an I-contract with probability λ and renewal of the current C-contract with probability $1 - \lambda$.

3. Permanent termination of the current GP upon project failure.

where $\lambda \in (0, 1]$ solves $\lambda(1 + \Delta)p_C m_C = [1 - (1 + \Delta)p_I]M_I$.

In hierarchical equilibria, new GPs are offered C-contracts with low agency rent and will be promoted to I-contracts with high agency rent upon her success. GPs with I-contract will be demoted to C-contract when they fail, and will be terminated if they fail under the C-contract. Intuitively, when the discount rate is high $\beta \geq p_I(1 + \Delta)$, the LP cares about the discounted future replacement costs. To save the I-contract replacement costs, instead of replacing those failed GPs out, the LP downgrades them to operate a C-contract, making I-contract more attractive to GPs. Promoting successful C-contract GPs to the more attractive I-contract increases the their promised continuation value upon success. Given no I-contract GP will be kicked out, this promotion and demotion feature redistributes the continuation values among GPs, providing extra incentive for GPs to exert efforts. Proposition 2 describes equilibria since for any $m_C \geq m_C^*$, the LP essentially offers a measure $m_C - m_C^*$ of C-contract characterized in proposition 1, and the LP is indifferent between different choices of $m_C \geq m_C^*$ because she receives zero profit from additional C-projects.

Equilibria with hierarchical contracts are the plausible because in reality, the probability for innovative projects to succeed under managerial effort is still very small, whereas the discount rate is unlikely to be lower than that. However, in the unlikely scenario where unicorns are commonplace, we have a separation of contracts:

Proposition 3 (Equilibria with Parallel Contracts)

When the probability for the innovative project to succeed under managerial effort is big, i.e., $p_I(1 + \Delta) < \beta$, the LP offers a measure $M_I(1 + \Delta)p_I$ of I-contracts to GPs who are recently successful under I-contracts and a measure $M_I(1 - (1 + \Delta)p_I)$ of to new GPs, all with terms:

1. $\alpha^I = \frac{(1-\beta p_I)e}{\beta \Delta p_I \rho X_I}$.

2. *Renewal of the same contract upon project success and payoff X_I.*

3. *Permanent termination of the current GP upon project failure.*

The LP offers an arbitrary measure $m_C \geq 0$ of C-contracts. To be more specific, she offers a measure of $m_C(1+\Delta)p_C$ to GPs who are recently successful under C-contracts and a measure of $m_C(1 - (1 + \Delta)p_C)$ to new GPs, all with terms:

1. $\alpha^C = \frac{(1-\beta p_C)e}{\beta \Delta p_C \rho X_C}$.

2. *Renewal of the same contract upon project success and payoff X_C.*

3. *Permanent termination of the current GP upon project failure.*

Intuitively, when the discount rate is low $\beta < p_I(1 + \Delta)$, the future replacement is not very costly for the LP. She prefers to replace the incumbent GPs regardlessly, suggesting that GPs are indifferent between I- and C-contracts. In the equilibrium, I- and C-contracts can be viewed as independent contracting problems and the optimal solution is similar to the fixed technology case.

The General Case

Given the results in the scenario with unlimited supply of zero profit C-projects, we are now ready to characterize the equilibrium under assumption 2 and 3. The following two propositions state that the equilibrium is essentially the same as the corresponding one in proposition 2 and 3.

Proposition 4 (Equilibrium with Hierarchical Contracts)
When the probability for the innovative project to succeed under managerial effort is small, i.e., $\beta \geq p_I(1 + \Delta)$, the LP offers a measure ϕ of I-contracts to GPs who are recently successful with terms:

1. $\alpha^I = \frac{(1-\beta^2 p_I)e}{\beta \Delta p_I \rho X_I}$.

2. *Renewal of the same contract upon project success and payoff X_I.*

3. *Continued funding under a C-contract upon project failure.*

The LP offers a measure $1 - \phi$ of C-contracts. To be more specific, she offers a measure $(1 - \phi)(1 - (1 + \Delta)p_C)$ of C-contracts to new GPs, $(1 - \phi)(1 - \lambda)(1 + \Delta)p_C$ to GPs who are recently successful under C-contracts, and $\phi(1 - (1 + \Delta)p_I)$ to GPs who recently failed under I-contracts, all with terms:

1. $\alpha^C = \frac{(1 - \beta(1 + \lambda\beta)p_C)e}{\beta\Delta p_C \rho X_C}$.

2. *Upon project success and payoff X_C, continued funding with an I-contract with probability λ and renewal of the current C-contract with probability $1 - \lambda$.*

3. *Permanent termination of the current GP upon project failure.*

where $\lambda > 0$ solves $\lambda(1 + \Delta)p_C(1 - \phi) = [1 - (1 + \Delta)p_I]\phi$.

In the unlikely scenario where $p_I(1 + \Delta) < \beta$, we have a separation of contracts:

Proposition 5 (Equilibrium with Parallel Contracts)

When the probability for the innovative project to succeed under managerial effort is big, i.e., $p_I(1 + \Delta) < \beta$, the LP offers a measure $\phi(1 + \Delta)p_I$ of I-contracts to GPs who are recently successful under I-contracts and a measure $\phi(1 - (1 + \Delta)p_I)$ to new GPs, all with terms:

1. $\alpha^I = \frac{(1 - \beta p_I)e}{\beta\Delta p_I \rho X_I}$.

2. *Renewal of the same contract upon project success and payoff X_I.*

3. *Permanent termination of the current GP upon project failure.*

The LP offers a measure $1 - \phi$ of C-contracts. To be more specific, she offers a measure $(1 - \phi)(1 + \Delta)p_C$ to GPs who are recently successful under C-contracts and a measure $(1 - \phi)(1 - (1 + \Delta)p_C)$ to new GPs, all with terms:

1. $\alpha^C = \frac{(1 - \beta p_C)e}{\beta\Delta p_C \rho X_C}$.

2. *Renewal of the same contract upon project success and payoff X_C.*

3. *Permanent termination of the current GP upon project failure.*

4 Model Implications

4.1 Persistence in Fund Performance and Manager Compensation

In our model, luck have an enduring impact on fund performance. The conventional interpretation of fund performance persistence as the evidence to support differential managerial skill stems from the idea that luck is independent over time. However, our model shows that even though luck is not persistent, in the optimal contract the LP implicitly rewards successful funds by giving a higher continuation value, which can be interpreted as higher probability of investing in rookie GP's next fund and possibly better contract terms. One period temporary luck can have a persistent impact through the promised continuation value channel as the optimal way to address the agency problem.

The capital flow channel along may not affect the persistence of fund gross performance. Suppose the GP finances the same type of ideas regardless of last period outcome, then conditional on GP exerting effort, the expected fund gross return is a constant across different funds and there would be no fund gross performance persistence. The endogenous deal flow affects gross performance persistence through its complementarity with capital flow. Under more favorable contract terms, the GP is more tolerant for failures and is willing to nurture ideas with innovative technology. Taking that into account, entrepreneurs with innovative ideas will take funding offers from I-contracts GPs. The complementarity between capital and deal flows suggest that GPs with more favorable contract terms generate an expected project payoff $(1 + \Delta)p_I X_I > (1 + \Delta)p_C X_C$, perform higher gross returns.

If the benefit of innovation is sufficiently large, the persistence applies to fund net fee performance as well. In the equilibrium, since $V_s^I - V_f^I = \beta \frac{e}{\Delta} < \frac{e}{\Delta} \leq V_s^C - V_f^C$, and $p_I < p_C$, GPs enjoy a larger amount of expected cash reward with I-contracts. On the other hand, nurturing a innovative idea generate higher gross return. If

$$\frac{p_I X_I}{p_C X_C} > \frac{1 - \alpha_C}{1 - \alpha_I}, \tag{23}$$

then the LP's profit exhibits $(1 + \Delta)p_I(1 - \alpha_I)\rho X_I > (1 + \Delta)p_C(1 - \alpha_C)\rho X_C$. When the

benefit of innovation dominates the agency cost for a more favorable contract for the GP, the model also predicts the persistence of fund net fee performance.

One critique for skill heterogeneity as an explanation for persistent performance is that more skilled GPs can charge a higher fee, thus eroding superior returns to the LP. Our theory survives this critique because GPs do not have differential skills and if they ask for higher fees, the LP can simply replace them by someone from the pool of aspirant GPs. That said, from Propositions 2 and 3, it naturally follows that in equilibrium GPs at funds with I-contracts are persistently better compensated, even though they do not have superior skills relative to those at funds with C-contracts: $V_{GP}^C = \frac{e}{\Delta}$, $V_{GP}^I = \frac{e}{\Delta}(1 + \beta)$. Moreover, depending on the parameter range, when a fund under I-contract fails, the LP may let the GP continue where as she has zero tolerance for failures for GPs in funds with C-contracts, i.e., $V_f^I = 0$.

4.2 Endogenous Capital and Deal Flows

To focus on contracting between the LP and GPs, in this paper we abstract away from fund size and the model has no explicit fund flow predictions. However, our model generates implicit fund flow predictions through the continuation value. In the equilibrium, the LP implicitly rewards successful funds by giving a higher continuation value, which can be interpreted as higher probability of investing in the GP's next fund and higher probability of offering better contract terms. Investing the GP's next fund suggests no fund outflow, and in the real world one way to offer a more favorable contract to the GP is to increase the fund size. This continued capital for funds or GPs that are recently successful is consistent with empirical findings in Kaplan and Schoar (2005).

Our model also predict that more innovative projects naturally flow to more innovative funds because either the more innovative entrepreneurs choose funds with I-contracts, or funds with I-contracts can select innovative projects. This is apparant from the assortative matching. This result is not an artifact of our binary technology or project type. Suppose we allow type θ EN to have probability of θ of having I-project, and suppose it costs an additional I to use I-technology. On the one hand, a type-θ enterpreneur would get $(1-\rho)[(1+$

$\Delta)p(\theta X_I + (1-\theta)X_C)]$ under innovative nurturing, which is more than $(1-\rho)(1+\Delta)pX_C$ under conventional nurturing. Hence she strictly prefers funds under I-Contracts. On the other hand, a fund with I-contract gets $\rho[(1+\Delta)p(\theta X_I + (1-\theta)X_C)] - I$ and therefore strictly prefers higher θ. That said, if the LP expects the fund to get project of expected quality $\theta < \underline{\theta} \equiv \frac{I}{\rho(1+\Delta)p(X_I - X_C)}$, she would rather use C-Contract.

Therefore, if the measure of funds with I-contracts in a period is $M_I \leq 1$, we have a unique matching equilibrium that is positive assortative in the sense that more innovative projects are matched with more innovative funds. All I-contracts are matched, and the remaining projects are matched to C-contracts. The average project quality for funds with I-contracts is $\theta_h \equiv \int_{F^{-1}(1-M_I)}^{\overline{\theta}} \theta dF(\theta)$. However, if $M_I > 1$, some I-contracts are also left unmatched.

More generally, we can allow for various types of innovative nurturing technology, K_1, K_2, etc. Type K_i augments the payoff to $X_I = K_i X_C$ and costs $\frac{IX_C^2}{(X_I - X_C)^2}(K_i - 1)^2$. The payoff on type θ is then $\theta K_i X_C + (1-\theta)X_C - \frac{IX_C^2}{(X_I - X_C)^2}(K_i - 1)^2$, which is super-modular in K_i and θ. Therefore, in equilibrium, it has to be positive assortative matching: more innovative technology matched with more innovative entrepreneur.

One critique for differential manager skills as an explanation for performance persistent is that if the fund size grows and investment has diminishing returns to scale, we would expect superior performance to be eroded. However, with the endogenous deal flow, this effect is mitigated and it is possible that even when fund size grows, an initially lucky fund can outperform for a long time.

4.3 Entrepreneurs' Contracts and Financing Choice

Another important evidence supporting differential managerial skill is the entrepreneur's funding offer choice. Hsu (2004) shows that entrepreneurs are more likely to accept funding offers from lead VC funds even with less favorable terms. Consistent with Hsu (2004), our model predicts entrepreneurs' preference on top VC funds. Given the fixed $\rho : 1-\rho$ contracts, entrepreneur with innovative ideas always prefer to take offers from I-contracts GPs because those funds will nurture their ideas in the efficient innovative way. Moreover, they will still

prefer to take offers from I-contracts GPs with less favorable term $\rho_I > \rho$ if

$$(1 + \Delta)p_I(1 - \rho_I)X_I > (1 + \Delta)p_C(1 - \rho)X_C. \tag{24}$$

This prediction suggests that entrepreneurs may be willing to accept a smaller share if the GP can efficiently nurture the project, generating larger expected project return. There could also be externalities among innovative or quality projects, if we allow each fund to finance multiple projects, which could further strengthen our results.

The analysis above implicitly assumes that entrepreneurs know the type of ideas they have, and entrepreneurs with conventional ideas would be indifferent among GPs. However, the endogenous deal flow in the equilibrium is robust if project quality is private information to the managers. Even if entrepreneurs may not know the quality of their ideas, always accepting funding offers from top VC funds is a weakly dominating strategy. Moreover, this extension predicts a VC certification effect. Matching up with a successful fund is a strong signal for project quality, and the entrepreneur and/or outsiders will update their beliefs about the idea accordingly. Different from standard certification stories based on high quality managers, here top-performing funds have an endogenous certification effect even though managers do not have differential skills.

4.4   Motivating Innovation

We also find that top-performing funds are more tolerant towards experimentation and encourage greater innovation. Different from Tian and Wang (2014), which implicitly treat VC fund's tolerance for failure as its characteristic, in this paper GPs show different attitude towards failure because they are given different contracts and accordingly receive different type of projects. There are two channels to make successful GPs more tolerant for failure. One reason is that in our model, higher agency rent is associated with higher continuation value given project failure ($V_f^I = \frac{e}{\Delta} > 0 = V_f^C$). This implies that:

$$V_s^I - V_f^I = \beta\frac{e}{\Delta} < \frac{e}{\Delta} = V_s^C - V_f^C. \tag{25}$$

23

So the difference between success and failure continuation value shrinks. Similar to Manso (2011), less punishment on failure motivates innovation. Another reason lies in the fact that the more favorable I-contract emphasize more on cash payment:

$$\beta(1+\Delta)p_I\alpha_I X_I > \beta(1+\Delta)p_C\alpha_C X_C. \tag{26}$$

The high cash payment motives GPs to implement innovative ideas with high expected return innovative nurturing technology.

4.5   Inter-contract Incentive Provision

Suppose there is no innovative ideas, if the LP offers the optimal contract conditional on only conventional ideas are available and makes zero profit, then she should be indifferent between investing in funds or not. As shown in the model solution, when both conventional and innovative projects present, the LP is strictly better off by investing in a non-zero measure set of conventional deals. The benefit comes from the fact that in the optimal contract, the LP can save motivating cost by redistributing continuation values among different GPs. The existence of another type of ideas enables the LP to redistribute continuation values among different type of contracts, creating extra incentives for GPs to exert efforts. In the case of conventional deals, the optimal contract without innovative ideas features $V_s^C = V_{GP}^C = \frac{e}{\Delta}$, while the optimal contract with innovative ideas features $V_s^C = V_{GP}^I > \frac{e}{\Delta}$, suggesting a lower α_C and strictly positive profit for C-fund investment. Moreover, it is straightforward to see that even if the LP loses money in fund investment when only conventional ideas are available, she is willing to invest in conventional deals when both type of deals are available and the loss is dominated by the benefit of motivating cost saving.

5   Conclusion

Persistence in fund performance in the private equity industry is often interpreted as evidence of differential abilities among the managers. We present a dynamic model of delegated

investments that produces performance persistence and predictability without managerial skill heterogeneity. Funding with terms condusive to innovation and quality projects exhibit strong complementarity, and endogenously flow to recently successful managers due to assortative matching and incentivization of managerial effort with continuation value. Initial luck therefore has an enduring impact on fund performance and managerial compensation, and investors benefit from working with multiple funds with tiered contracts. Consistent with empirical findings, our model also predicts that venture funds that persistently outperform encourage greater innovation and attract quality projects with seemingly less favorable contract terms.

References

Acharya, Viral V, Oliver F Gottschalg, Moritz Hahn, and Conor Kehoe, 2013, Corporate governance and value creation: Evidence from private equity, *Review of Financial Studies* 26, 368–402.

Berk, Jonathan B., and Richard C. Green, 2004, Mutual fund flows and performance in rational markets., *Journal of Political Economy* 112, 1269–1295.

Biais, Bruno, Thomas Mariotti, Guillaume Plantin, and Jean-Charles Rochet, 2007, Dynamic security design: Convergence to continuous time and asset pricing implications, *The Review of Economic Studies* 74, 345–390.

Bolton, Patrick, and David S Scharfstein, 1990, A theory of predation based on agency problems in financial contracting, *The American economic review* pp. 93–106.

DeMarzo, Peter M, and Michael J Fishman, 2007, Optimal long-term financial contracting, *Review of Financial Studies* 20, 2079–2128.

Ewens, Michael, and Matthew Rhodes-Kropf, 2015, Is a vc partnership greater than the sum of its partners?, *The Journal of Finance* 70, 1081–1113.

Gale, David, and Lloyd S Shapley, 1962, College admissions and the stability of marriage, *The American Mathematical Monthly* 69, 9–15.

Glode, Vincent, and Richard C. Green, 2011, Information spillovers and performance persistence for hedge funds., *Journal of Financial Economics* 101, 1–17.

Gompers, Paul, Anna Kovner, Josh Lerner, and David Scharfstein, 2006, Skill vs. luck in entrepreneurship and venture capital: Evidence from serial entrepreneurs, Discussion paper, National bureau of economic research.

Gompers, Paul A, 1996, Grandstanding in the venture capital industry, *Journal of Financial economics* 42, 133–156.

Harris, Robert S., Tim Jenkinson, Steven N. Kaplan, and Rudiger Stucke, 2014, Has persistence persisted in private equity? evidence from buyout and venture capital funds., *Working Paper*.

He, Zhiguo, 2009, Optimal executive compensation when firm size follows geometric brownian motion, *Review of Financial Studies* 22, 859–892.

Hochberg, Yael V., Alexander Ljungqvist, and Annette Vissing-Jorgensen, 2014, Informational holdup and performance persistence in venture capital., *Review of Financial Studies* 27, 102–152.

Hsu, David H., 2004, What do entrepreneurs pay for venture capital affiliation?, *The Journal of Finance* 59, 1805–1844.

Kaplan, Steven N, and Antoinette Schoar, 2005, Private equity performance: Returns, persistence, and capital flows, *The Journal of Finance* 60, 1791–1823.

Manso, Gustavo, 2011, Motivating innovation., *The Journal of Finance* 66, 1823–1860.

Marquez, Robert, Vikram Nanda, and M. Deniz Yavuz, 2015, Private equity fund returns and performance persistence., *Review of Finance* 19, 1783–1823.

Megginson, William L, and Kathleen A Weiss, 1991, Venture capitalist certification in initial public offerings, *The Journal of Finance* 46, 879–903.

Phalippou, Ludovic, and Oliver Gottschalg, 2009, The performance of private equity funds, *Review of Financial Studies* 22, 1747–1776.

Sannikov, Yuliy, 2008, A continuous-time version of the principal-agent problem, *The Review of Economic Studies* 75, 957–984.

Shleifer, Andrei, and Robert W Vishny, 1997, The limits of arbitrage, *The Journal of Finance* 52, 35–55.

Sorensen, Morten, 2007, How smart is smart money? a two-sided matching model of venture capital., *The Journal of Finance* 62, 2725–2762.

Stiglitz, Joseph E, and Andrew Weiss, 1983, Incentive effects of terminations: Applications to the credit and labor markets, *The American Economic Review* 73, 912–927.

Tian, Xuan, and Tracy Yue Wang, 2014, Tolerance for failure and corporate innovation., *Review of Financial Studies* 27, 211–255.

Appendix

A Derivations and Proofs

A.1 Proof of Proposition 1

Proof. We know payment to the GP has value $V_{GP}^C = \frac{e}{\Delta} + \beta V_f^C$. Suppose in the equilibrium contract V_f^C involves positive probability of continuation with a renewed contract (potentially of different α and renewal policy but same technology.), the LP is better off offering a similar contract but with $V_f^C = 0$ (termination) and the corresponding α based on equation 9, and giving the same continuation contract that would be given to the current GP in the original equilibrium to a new GP from the aspirant pool. In terms of the total payments to GPs, this costs the LP less as in equilibrium α is lower, and the stream of payments to the LP is higher than in the original equilibrium because GPs are still motivated to exert effort, and the LP gets a bigger share. So the original contract form cannot be optimal. Therefore, it has to be $V_f^C = 0$.

Next, we notice that if V_s^C does not involve continuation for sure, consider the deviation by the LP: in the states that she would have terminated the GP and offers a replacement contract to a new GP from the pool under the original equilibrium, now she allows the current GP to continue with the same terms as in the replacement contract. The current GP is happy to accept, now the LP can profitably deviate again by lowering α based on equation 9 because V_s^C is higher. The stream of payments to the LP is now higher while the effort motivation cost is still the same in each period. Therefore an optimal contract form involves continuation upon success for sure.

It remains to show that in equilibrium, we do not need different contracts with the same technology. Suppose we have a distribution of different C contracts, we note that all the payoffs are linear in α, so we can simply use the mean α to be our equilibrium contract and the payoffs would be equivalent. In other words, we can use one single type of C-contract.

The case of using I-Contract alone in equilibrium is the same.

□

A.2 Proof of Lemma 1

Proof. Suppose in equilibrium, the LP offers a measure m_C^1 of C-contract $C^1 = \{\alpha^1, V_f^{C,1}, V_s^{C,1}\}$ and a measure m_C^2 of C-contract $C^2 = \{\alpha_C^2, V_f^{C,2}, V_s^{C,2}\}$. When $V_{GP}^{C,1} = V_{GP}^{C,2}$, then those two types of contracts can be considered as a measure $m_C^1 + m_C^2$ of weighted contract $C = \frac{m_C^1}{m_C^1+m_C^2}C^1 + \frac{m_C^2}{m_C^1+m_C^2}C^2$, where any promised future contracts C^1 and C^2 are replaced by C. When $V_{GP}^{C,1} \neq V_{GP}^{C,2}$, without loss of generality, assume $V_{GP}^{C,1} < V_{GP}^{C,2}$, then the LP can be better off by replacing C^2 with C^1 and changing any promised future contracts C^2 to C^1. By construction, this is still a steady state. Similarly, in equilibrium the LP offers only one type of I-contracts.

Since $\beta \Delta p_C \rho X_C = (\frac{1}{\beta} - p_C(1+\Delta))e$, the LP is indifferent to finance or abandon C-projects under the contract characterized in proposition 1. $p_C X_C < p_I X_I$ suggests that the LP earns strictly positive profit by offering a contract similar to the one in proposition 1. Now suppose in equilibrium $m_I < M_I$, then the LP can always offer a measure $M_I - m_I$ of I-contract similar to the one in proposition 1, a contradiction. □

A.3   Proof of Lemma 2

Proof. Suppose in equilibrium, there is a measure M_I of I-contracts and a measure m_C^1 of C-contracts C^1, then all I-projects must be financed by I-contracts, otherwise adding I-contracts as in Proposition 1 would be a profitable deviation by the LP. Now we prove the lemma in two steps. We first argue that upon seeing failure (zero output) under C-contract, the LP finds it suboptimal to let the GP continue with C-contract. We then show that it would not be optimal to let the GP continue in with I-contract either.

Suppose after a failure with C-contract C^1, the probabilities of getting I- and C- contracts are $\pi_I \geq 0$ and $\pi_C > 0$. Then we have,

$$V_f^{C^1} = (1 - \pi_I - \pi_C)0 + \pi_I V_{GP}^I + \pi_C V_{GP}^{C^1}$$
$$V_{GP}^{C^1} = \frac{e}{\Delta} + \beta V_f^{C^1} = \frac{e}{\Delta} + \pi_I \beta V_{GP}^I + \pi_C \beta V_{GP}^{C^1}$$

which gives the total payment to GP under C-contract when it is matched with a project:

$$V_{GP}^{C^1} + e = \frac{\frac{e}{\Delta} + \beta \pi_I V_{GP}^I}{1 - \pi_C \beta} + e > \frac{e}{\Delta} + e. \tag{27}$$

Consider an alternative scenario, where we adding a measure $m_2^C > 0$ of C-projects under the contract characterized in proposition 1, denoted by C^2. Given $\beta \Delta p_C X_C \rho = (\frac{1}{\beta} - p_C(1 + \Delta))e$ and proposition 1, this is still a steady state and the LP receives the same profit. Given lemma 1, we construct a measure $m_C^1 + m_C^2$ of weighted contract $C = \frac{m_C^1}{m_C^1 + m_C^2}C^1 + \frac{m_C^2}{m_C^1 + m_C^2}C^2$, where any promised future contracts C^1 and C^2 are replaced by C. By construction, this is still a steady state. Then we have,

$$V_f^C = (1 - \frac{m_C^1}{m_C^1 + m_C^2}\pi_I - \frac{m_C^1}{m_C^1 + m_C^2}\pi_C)0 + \frac{m_C^1}{m_C^1 + m_C^2}\pi_I V_{GP}^I + \frac{m_C^1}{m_C^1 + m_C^2}\pi_C V_{GP}^C$$
$$V_{GP}^C = \frac{e}{\Delta} + \beta V_f^C = \frac{e}{\Delta} + \frac{m_C^1}{m_C^1 + m_C^2}\pi_I \beta V_{GP}^I + \frac{m_C^1}{m_C^1 + m_C^2}\pi_C \beta V_{GP}^C$$

which gives the total payment to GP under C-contract when it is matched with a project:

$$
\begin{aligned}
V_{GP}^C + e &= \frac{\frac{e}{\Delta} + \beta \frac{m_C^1}{m_C^1 + m_C^2}\pi_I V_{GP}^I}{1 - \frac{m_C^1}{m_C^1 + m_C^2}\pi_C \beta} + e \\
&= \frac{m_C^1}{m_C^1 + m_C^2}(\frac{\frac{e}{\Delta} + \beta \pi_I V_{GP}^I}{1 - \frac{m_C^1}{m_C^1 + m_C^2}\pi_C \beta} + e) + \frac{m_C^2}{m_C^1 + m_C^2}(\frac{\frac{e}{\Delta}}{1 - \frac{m_C^1}{m_C^1 + m_C^2}\pi_C \beta} + e) \\
&< \frac{m_C^1}{m_C^1 + m_C^2}(\frac{\frac{e}{\Delta} + \beta \pi_I V_{GP}^I}{1 - \pi_C \beta} + e) + \frac{m_C^2}{m_C^1 + m_C^2}(\frac{e}{\Delta} + e) \\
&= \frac{m_C^1}{m_C^1 + m_C^2}(V_{GP}^{C^1} + e) + \frac{m_C^2}{m_C^1 + m_C^2}(V_{GP}^{C^2} + e)
\end{aligned}
\tag{28}
$$

Also, since $V_{GP}^{C^2} = \frac{e}{\Delta} < V_{GP}^C$, $V_{GP}^C < V_{GP}^{C^2}$, and V_{GP}^I decreases given the contract V_{GP}^C. Thus compared to

the two contracts C^1 and C^2, the LP pays less agency costs under contract C and is strictly better off. The total cost paid to all the GPs under C contracts is

$$C = m_C \left[V_{GP}^C + e + \frac{\beta}{1-\beta}(1 - p_C(1+\Delta))(1 - \pi_C - \pi_I^{'})(V_{GP}^C + e) \right] \tag{29}$$

where $m_C = m_C^1 + m_C^2$, $\pi_I^{'} = \frac{m_C^1}{m_C^1 + m_C^2}\pi_I$, and C_I is the total payments to GPs under I-contracts. Then $\frac{\partial C}{\partial \pi_C}$ is

$$\frac{\partial}{\partial \pi_C}\left[\frac{\frac{e}{\Delta} + \beta\pi_I^{'}V_{GP}^I}{1 - \pi_C\beta} + \frac{\beta}{1-\beta}(1 - p_C(1+\Delta))(1 - \pi_C - \pi_I^{'})\left(\frac{\frac{e}{\Delta} + \beta\pi_I^{'}V_{GP}^I}{1 - \pi_C\beta} \right) \right]$$

$$= \beta\left(\frac{e}{\Delta} + \beta\pi_I^{'}V_{GP}^I \right)\left[\frac{1}{(1 - \pi_C\beta)^2} + \frac{\beta}{1-\beta}(1 - p_C(1+\Delta))\left(\frac{1 - \pi_C - \pi_I^{'}}{(1 - \pi_C\beta)^2} - \frac{1}{\beta(1 - \pi_C\beta)} \right) \right]$$

$$= \frac{\beta}{(1 - \pi_C\beta)^2}\left(\frac{e}{\Delta} + \beta\pi_I^{'}V_{GP}^I \right)\left[1 + \frac{\beta}{1-\beta}(1 - p_C(1+\Delta))(1 - \pi_C - \pi_I^{'} - (\frac{1}{\beta} - \pi_C)) \right]$$

$$= \frac{\beta}{(1 - \pi_C\beta)^2}\left(\frac{e}{\Delta} + \beta\pi_I^{'}V_{GP}^I \right)\left[p_C(1+\Delta) - \frac{\beta}{1-\beta}\pi_I^{'}(1 - p_C(1+\Delta)) \right] \tag{30}$$

which is positive for sufficiently small $\pi_I^{'}$. So unless $\pi_C = 0$, we can always find a large enough m_C^2 such that there is a better LP strategy. Therefore, $\pi_C = 0$ and $V_f^C = \pi_I V_{GP}^I$.

Now, suppose $\pi_I > 0$ in the stationary equilibrium, then there must be a corresponding measure $\pi_I m_C(1 - (1+\Delta)p_C)$ of contemporaneous GPs under I-contract who are not renewed with I-contract. On the one hand, if these GPs are successful under I-contract, consider the alternative I'- and C'-contracts with all the terms intact except for $\pi_I^{'} = \pi_I - \epsilon$ and V_s^I has $\frac{m_C}{M_I}\epsilon$ probability higher of renewing with I-contracts, where $\epsilon > 0$ is an infinitesimal deviation. This would make the transition balanced and would still motivate efforts in both contracts yet reducing the motivation cost for C'-contracts. On the other hand, if successful GPs under I-contracts are all renewed I-contract, at most a measure $M_I(1 - (1+\Delta)p_I)$ of GPs under I-contract are not renewed I-contract, because the LP always has the option to add more C-projects, $M_I(1 - (1+\Delta)p_I)$ can be less the measure of successful GPs under C-contracts. This implies that the LP can use C'-contracts with all the terms intact except for $\pi_I^{'} = \pi_I - \epsilon$ and V_s^C has $\frac{\epsilon(1-\phi)}{\phi}$ probability higher of continuing with I-contracts, which is again a profitable deviation. Therefore $\pi_I = 0$. $\square$

A.4   Proof of Lemma 3

Proof. Again, we prove the lemma in two steps. First we show that upon failing under I-contract, the agent would not be renewed with I-contract. Now suppose V_f^I involves positive probability of continuing with I-contracts, I-projects are sufficiently scarce that we can instead let all GPs that should receive I-contracts after their I-projects failures change to C-contracts, and since in equilibrium no I-projects are financed by funds with C-contracts, we can let some GPs who should be given C-contracts after C-project successes to receive I-contracts instead. This swap is feasible given the fact that the LP can add sufficiently many C-projects and $0 < \Delta < \frac{1-p_C}{p_C}$. The transition is still balanced, but we can still motivate efforts with reduced costs as V_f^I is reduced. Therefore, it cannot be the case that upon failure under I-contract, the GP still continues with I-contract.

Next, let $V_f^I = \pi_C V_{GP}^C$, we get $V_{GP}^I = (1 + \beta\pi_C)\frac{e}{\Delta}$. Then $\frac{\partial C_I}{\partial \pi_C}$ can be written as

$$
\begin{aligned}
&\beta\frac{e}{\Delta} + \frac{\beta}{1-\beta}\frac{\partial}{\partial \pi_C}\left[(1 - p_I(1+\Delta))(1 - \pi_C)\frac{e}{\Delta}\right] \\
&= \beta\frac{e}{\Delta}\left(1 - \frac{1 - (1+\Delta)p_I}{1-\beta}\right)
\end{aligned}
\tag{31}
$$

which is positive if and only if $p_I(1 + \Delta) > \beta$. When $p_I(1+\Delta) \leq \beta$, $\frac{\partial C_I}{\partial \pi_C} \leq 0$. In the equilibrium $\pi_C = 1$, then $V_f^I = V_{GP}^C$ and $V_{GP}^I = (1+\beta)\frac{e}{\Delta}$. When $p_I(1+\Delta) > \beta$, $\frac{\partial C_I}{\partial \pi_C} > 0$. In the equilibrium $\pi_C = 0$, then $V_f^I = 0$ and $V_{GP}^I = \frac{e}{\Delta}$. It then follows $V_s^I = V_{GP}^I$. $\qquad\square$

A.5 Proof of Proposition 2

Proof. Given the facts that the LP can add sufficiently many C-projects, $0 < \Delta < \frac{1-p_C}{p_C}$ and $m_c \geq m_C^*$, there are more GPs who are recently successful under C-contracts than GPs who recently failed under I-contracts. Let $\lambda = \frac{[1-(1+\Delta)p_I]M_I}{(1+\Delta)p_C m_c}$, lemma 2 and 3 determines the transition of GPs in equilibrium.

Given the equilibrium transition of GPs, $V_f^C = 0$, so $V_{GP}^C = \frac{e}{\Delta} + \beta V_f^C = \frac{e}{\Delta}$. Similarly, $V_f^I = V_{GP}^C = \frac{e}{\Delta}$ and $V_{GP}^I = \frac{e}{\Delta} + \beta V_f^I = (1+\beta)\frac{e}{\Delta}$. Thus $V_s^I = V_{GP}^I$ and α^I is pinned down by equation 10. Similarly, α^C is determined by equation 8 and $V_s^C = (1 - \lambda)V_{GP}^C + \lambda V_{GP}^I = (1 + \lambda\beta)\frac{e}{\Delta}$. $\qquad\square$

A.6 Proof of Proposition 3

Proof. Given lemma 2 and 3, the transition of GPs is straight forward and feasible. Given the equilibrium transition of GPs, $V_f^C = V_f^I = 0$, so $V_{GP}^C = \frac{e}{\Delta} + \beta V_f^C = \frac{e}{\Delta} = V_{GP}^I$. Thus $V_s^I = V_{GP}^I = V_s^C = V_{GP}^C$. α^C and α^I are determined by equation 8 and 10, respectively. $\qquad\square$

A.7 Proof of Proposition 4 and 5

Proof. Given the assumption 3, all project are profitable and the LP can at least finance projects with the contracts stated in proposition 1, suggesting that in any equilibrium all projects are financed. Conditional on all projects being financed, the optimal contracting problem can be transfered to a agency costs minimizing problem.

In proposition 2, the optimal contract for each $m_C \geq m_C^*$ minimizes the associated agency costs given the financed project type ratio $\frac{m_C}{M_I}$, which is independent of the expected project payoff. Given assumption 2, for each ratio $\frac{1-\phi}{\phi}$, there is a corresponding $\frac{m_C}{M_I}$ and the equilibrium for the corresponding m_C in 2 is the equilibrium for the project distribution ϕ in 4. Similarly, one can prove the case for proposition 3 and 5. $\qquad\square$

B Overlapping Generations of GPs

We provides a micro-foundation for the one period commitment power assumption by introducing an overlapping generation model for GPs.

There is an infinite supply of venture capitalists aspirant of becoming general partners of VC funds. We refer to them generically as GPs. To capture the finite life-cycle of VCs who typically set up sequential

funds, we assume overlapping generations of GPs and they each lives for three periods.

In the first period, a new-born rookie GP can join force with a seasoned GP born in the previous period in order to setup a VC fund. After successful fundraising, the rookie GP provides labor while the seasoned GP contributes his network or other resources. We label the rookie GP g_r and the seasoned GP g_{se}, and the pair $g = (g_r, g_{se}) \in \mathcal{G}_t$. The fund can finance one representative project and g_r and g_{se} have to decide on whether to incur effort costs e_r and e_{se} respectively to augment the project-success probability by $0 < \Delta < \frac{1-p}{p}$.[10] The GPs also choose the nurturing technology based on their contracts with the investors.[11] The seasoned GP and the rookie GP then decide the way they share the fund's economic rents including both cash payment and LP's promised value in the next period. For simplicity, we assume that $e_r = e_{se} = e$, and GPs equally split their economic rents from running the fund. All funds require a start capital of I_0, and funds with innovative nurturing technology requires an additional capital I, which can be viewed as a provision in the contract for additional capital call on investors, or both financial and time costs for tolerating early failures and rewarding long-term success (see Manso (2011)), or bigger investment in startup team-building. Upon failed fundraising, the rookie GP pursues an alternative career to build up personal network and expertise, and aspires to start a fund again in the next period.

In the second period, for GPs that successfully raised fund in the previous period, project outcomes are realized, and they pay the entrepreneur and investors. The seasoned GP then gets his share of economic rent and retires, since this is his last period, he will be paid fully by cash while the rookie GP may get a combination of cash and continuation value. Regardless of the outcome of his fundraising in the first period, a rookie becomes seasoned and is able to set up a new fund with a new-born GP for fundraising. With failed fundraising, he prematurely retires, and with successful fundraising, he works for one more period and gets the payoff from running fund before retiring.

In this setup, the limited commitment emerges as a natural constraint for the LP because GPs are short-lived. In each round of fund, since both g_r and g_{se} face the same effort cost and they equally split the total economic rent, both agents are solving the same incentive-compatibility problem. The only difference is that while for rookie GP g_r the realization of economics rent is a combination of both cash payment and continuation value, seasoned GP g_{se} are compensated solely with cash.

---

[10] We can interpret the effort as screening out bad projects or actively contributing to the startup team.

[11] This can be interpreted as experimentation jointly with the entrepreneurs. GPs in general can influence firm operations, team building, and experimentation styles by setting different contract terms, taking hidden actions, showing different attitudes towards failures etc.